# OPTIMIZATION OF A GO$_2$/GH$_2$ SWIRL COAXIAL INJECTOR ELEMENT

**P. Kevin Tucker**
NASA
Marshall Space Flight Center, AL 35812

**Wei Shyy and Rajkumar Vaidyanathan**
Department of Aerospace Engineering, Mechanics & Engineering Science
University of Florida
Gainesville, Florida 32611

## ABSTRACT

An injector optimization methodology, *method i*, is used to investigate optimal design points for a gaseous oxygen/gaseous hydrogen (GO$_2$/GH$_2$) swirl coaxial injector element. The element is optimized in terms of design variables such as fuel pressure drop, $\Delta P_f$, oxidizer pressure drop, $\Delta P_o$, combustor length, $L_{comb}$, and full cone swirl angle, $\Theta$, for a given mixture ratio and chamber pressure. Dependent variables such as energy release efficiency, *ERE*, wall heat flux, $Q_w$, injector heat flux, $Q_{inj}$, relative combustor weight, $W_{rel}$, and relative injector cost, $C_{rel}$, are calculated and then correlated with the design variables. An empirical design methodology is used to generate these responses for 180 combinations of input variables. *Method i* is then used to generate response surfaces for each dependent variable. Desirabilitiy functions based on dependent variable constraints are created and used to facilitate development of composite response surfaces representing some, or all, of the five dependent variables in terms of the input variables. Two examples illustrating the utility and flexibility of *method i* are discussed in detail. First, joint response surfaces are constructed by sequentially adding dependent variables. Optimum designs are identified after addition of each variable and the effect each variable has on the design is shown. This stepwise demonstration also highlights the importance of including variables such as weight and cost early in the design process. Secondly, using the composite response surface that includes all five dependent variables, unequal weights are assigned to emphasize certain variables relative to others. Here, *method i* is used to enable objective trade studies on design issues such as component life and thrust to weight ratio.

## INTRODUCTION

In order to meet future launch program goals, the Spaceliner 100 Technology Roadmap[1] specifies very aggressive system goals for safety, life and cost per pound of payload launched into Earth orbit. Spaceliner 100 safety goals would decrease catastrophic events from the current 1 in 200 to 1 in 1,000,000 in 15 years. The life goal would be increased from the current 200 manned missions per year to 2000-5000 per year over the same time period. Concurrently, the

cost goal aims to reduce the cost of delivering payloads to Earth orbit from the current $10,000 per pound to $1000 per pound in 10 years and to $100 per pound in 15 years and ultimately to $10 per pound.

Design and development of advanced propulsion systems will be crucial to meeting these goals. Propulsion systems which meet these requirements must not only have high thrust to weight ratios, but also achieve higher operability and maintainability standards than in previous or current programs. Combustor designs, and injector designs in particular, will be key issues in meeting these goals. The injector design determines performance and stability, and is, therefore, the key factor governing injector face and chamber wall heat transfer/compatibility issues. Injector design also affects engine weight, cost, operability and maintainability.

The injector design methodologies used successfully in previous programs were typically based on large subscale databases and the empirical design tools derived from them[2,3,4,5,6]. These methodologies were often guided by extensive sub-and full-scale hot-fire test programs. Current and planned launch vehicle programs have relatively low budgets and aggressive schedules; neither of which is conducive to the large test programs of the past. Also, new requirements for operability and maintainability require that the injector design be robust. Also, the goal for increased robustness will require evaluation of a larger design space earlier in the design process. In the context of these new goals, development of broader and more efficient injector design methodologies seems to be a worthy pursuit.

This work demonstrates a new design methodology called *method i* [7,8,9](Methodology for Optimizing the Design of Injectors) which seeks to address the above issues in the context of injector design. Simply put, *method i* is used to generate appropriate design data and then guide the designer through the information toward an optimum design subject to his specified constraints. Since *method i* is structured so that any pertinent information source can be used, design data can be obtained from existing databases and empirical design methodologies. If required, new data can be generated with modern experimental techniques or appropriate CFD models.

As implied above, *method i* is comprised of two discrete entities. The first element is the tool used to generate the design data—in this work, an empirical design methodology for $GO_2/GH_2$ injectors generated by Calhoon et al.[10,11] The second element in *method i* is a group of optimization techniques. It is the optimization capability that extends *method i* beyond previous injector design methodologies. The optimization scheme allows large amounts of inter-related information to be managed in such a way that the extent to which variables influence each other can be objectively evaluated and optimal design points can be identified with confidence. In this work, the Response Surface Method (RSM)[12] is used to facilitate the optimization. The RSM approach is to conduct a series of well-chosen experiments (i. e., numerical, physical, or both) and use the resulting function values to construct a global approximation (i. e., response surface) of the measured quantity (i. e., response) over the design space. A standard constrained optimization algorithm is then used to interrogate the response surface for an optimum design.

The approach used to develop and demonstrate this new methodology is divided into three main tasks. Task 1 is a proof of concept where the basic methodology is developed and demonstrated

on single element injectors. Work on Task 1 for the shear coaxial and impinging elements has been reported previously.[7,8,9] The work for the impinging element, which completes the empirical database for Task 1, is presented below. To conclude Task 1, all the design data, along with optimization techniques developed to date, will be demonstrated in an element selection/preliminary design process.

Task 2 involves replacing/augmenting the empirical data with data from physical and numerical experiments (i.e., test data and validated CFD analyses). Task 3 involves using CFD analyses and empirical methods to design a multi-element injector consisting of 7-12 elements. Optimization will be done in the context of single element variables plus element pattern, element spacing, film cooling, etc.


## SCOPE OF CURRENT EFFORT

This paper presents the design optimization of a swirl coaxial injector element. The swirl coaxial element has been used for somewhat sparingly in this country, but has been widely used in Russia because of its reported ability to perform well over a large throttle range.[13] A schematic of the element is shown in Fig. 1. The empirical design methodology of Calhoon et al uses the oxidizer pressure drop, $\Delta P_o$, fuel pressure drop, $\Delta P_f$, combustor length, $L_{comb}$, and the full cone swirl angle, $\Theta$, as independent variables. Due to stability considerations for this injector design, the $\Delta P_o$ range is set to 10-20% of the chamber pressure, while the $\Delta P_f$ range is set to 2-20% of chamber pressure. The combustor length, defined as the distance from the injector to the end of the barrel portion of the chamber ranges from 2-8 inches. The full cone swirl angle is allowed to vary from 30-90°. The dependent variables modeled are *ERE* (a measure of element performance), wall heat flux, $Q_w$, injector heat flux, $Q_{inj}$, relative combustor weight, $W_{rel}$, and relative injector cost, $C_{rel}$.

In the following sections, the injector model and the generation of design data are briefly discussed. Response surfaces for each of the dependent variables are generated and then combined into a joint surface to facilitate the optimization process. Optimization of the element is demonstrated by applying equal weights for all dependent variables as they are added to the joint response surface one at a time and by applying unequal weights that might reflect specific design priorities and trades.


## SWIRL COAXIAL ELEMENT DESIGN MODEL

This section provides details of the models used to generate the design data for the dependent variables noted above. The process for generating the design data is described and sample results are also presented. The chamber pressure, mixture ratio, and propellant flow rates selected for this example are:

$$P_c = 1000\,psi$$

$$MR = 6$$

$$\dot{m}_{GO_2} = 0.25\,lb_m/sec$$

$$\dot{m}_{GH_2} = 0.042\,lb_m\,sec$$

The gaseous propellants are injected at a temperature of 540 R.

## Dependent Variable Models

Reference to Figure 1 shows that the $GO_2$, flowing in the center post of the element, exits the element with both radial and axial velocity components. This effect is achieved by introducing the $GO_2$ tangentially into the center post through small slots. When the $GO_2$ under hydrostatic head is forced through the tangential slots, part of the pressure head is converted into a velocity head, causing a rotational velocity in the element. With the operating conditions fixed at the above-noted levels, the work of Doumas and Laster[14] is used to define the element geometry required to generate $GO_2$ swirl angles. Although developed for liquids, this work has been used successfully to design swirl coaxial element for gaseous propellants.[15,16] For a specified $\Delta P_o$ and swirl angle, $\Theta$, the number and size of tangential slots, the discharge coefficient, the $GO_2$ center post diameter, $d_o$, and the $GO_2$ velocity components, $V_{or}$ and $V_{oa}$ are calculated. These quantities are then used to determine the dependent variables for each design condition.

Figure 1. Swirl Coaxial Injector Element Schematic
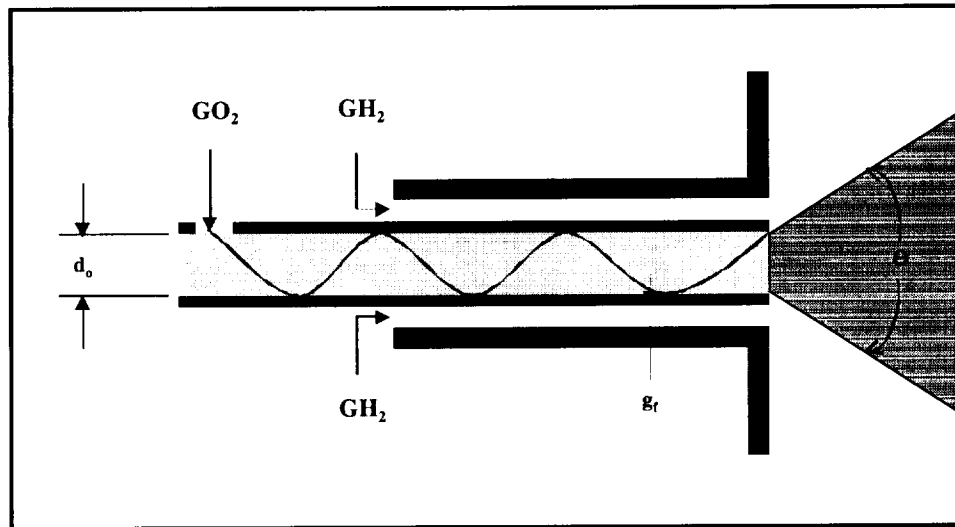


The element *ERE* is calculated according to the empirical design methodology of Calhoon et al. it is a function of all four independent variables noted above. A cold flow mixing efficiency at $\Theta=90°$, $E_{m,90}$, is correlated by:

$$E_{m,90} = 100 - 5 ln \left[ \frac{K_s}{L_{cold}/d_o} \right]$$ (1)

The cold flow mixing length, $L_{cold}$, is correlated from a known chamber length, $L_{comb}$. The $GO_2$ post diameter, $d_o$, is a function of $\Delta P_o$ and $\Theta$. Smaller values of $d_o$ correspond to large values of $\Delta P_o$ and smaller swirl angles. The empirical swirl factor, $K_s$, is a function of the normalized differential injection velocity, $(V_f-V_o)/V_o$. For fixed propellant mass flow rates, the velocities $V_o$ and $V_f$ are functions of their pressure drops across the injector, $\Delta P_o$ and $\Delta P_f$, respectively. For a given $\Delta P_o$, $V_o$ also depends on the swirl angle. Lower $V_o$'s are a product of higher swirl angles. Cold flow mixing is thereby enhanced with higher values of $V_o$ (i.e. $\Delta P_o$) and $L_{comb}$. Lower values of $V_f$ (i.e. $\Delta P_f$) and $\Theta$ also tend to enhance cold flow mixing.

A fractional factor, $f_s$, is applied to $E_{m,90}$ to account for the lower levels of cold flow mixing found with swirl angles less than 90°. The resultant measure of cold flow mixing, $E_{m,\Theta}$, is a product of $E_{m,90}$ and $f_s$. This factor, for a given design, is a function of the normalized differential injection velocity and the ratio of radial to axial $GO_2$ velocity, $V_{or}/V_{oa}$. Increasing values of both quantities increase $f_s$, with a value of $f_s = 1$ being found at $V_{or}/V_{oa} = 1$ ($\Theta=90°$) for all values of $(V_f - V_o)/V_o$. Larger values of $f_s$ increase cold flow mixing. These values are found at low $\Delta P_o$ and high $\Delta P_f$ and $\Theta$. There is no dependency of $f_s$ on chamber length. The trends for cold flow mixing are opposite those noted above. Finally, $ERE$ is proportional to $E_{m,\Theta}$. The effect of the competing influences of the independent variables on $ERE$ trends will be discussed later.

The wall heat flux $Q_w$, is correlated with the propellant momentum ratio as defined by:

$$MR = \frac{m_o u_o}{m_f u_f}$$ (2)

The wall heat flux curve from the Calhoon et al. Methodology is fairly flat, varying only about 10% from high to low for the range of pressure drops considered in this effort. $Q_w$ decreases with increasing $V_o$ (high $\Delta P_o$ and low $\Theta$) and decreasing $V_f$ (high $\Delta P_f$). That $Q_w$ would decrease with increasing $V_o$ is counter to intuition. It seems that high values of $V_o$, for any $\Theta$, would result in higher mixture ratios in the wall region as is the case for liquid $O_2$. This effect is not discussed by Calhoon et al.

The heat flux seen by the injector, $Q_{inj}$, is actually modeled by the distance from the injector at which the propellant streams intersect. This axial distance is measured at the radial position corresponding to the center of the coaxial fuel annulus. It is here that the streams begin to mix and burn. This measure is qualitative, but captures the trend that higher injector heat fluxes occur the nearer the injector that the combustion occurs. The axial distance is affected directly by the swirl angle, and, indirectly, by the propellant pressure drops. $Q_{inj}$ decreases with decreasing swirl angle, increasing $GO_2$ pressure drop and decreasing $GH_2$ pressure drop. Swirl angle has the largest effect, while $\Delta P_o$ is the least significant factor.

The relative combustor weight, $W_{rel}$, is simply a function of the combustor length, $L_{comb}$, the distance from the injector to the end of the barrel portion of the chamber. The longer the combustor, the more it weighs.

The relative injector cost, $C_{rel}$, is a function of the fuel gap width and the width of the tangential slots used to induce the swirl in the $GO_2$ center post. Larger values of both variables result in lower machining costs, and thus lead to lower injector cost. The fuel gap width increases with increasing $\Delta P_o$, and decreasing values of $\Delta P_f$ and $\Theta$. Swirl slot width increases with lower values of $\Delta P_o$ and $\Theta$. Overall, $C_{rel}$ decreases with increasing $\Delta P_o$ and decreasing $\Delta P_f$ and $\Theta$. Fuel pressure drop and swirl angle are the most significant factors.

## Generation of Design Data

The operating given above and independent variables (constrained to the previously noted ranges) are used to generate the design data for element optimization studies. A matrix of propellant pressure drop combinations was developed and nine combinations were selected for use in populating the design data base. There are 20 combinations of $L_{comb}$ and $\Theta$ for each $\Delta P$ combination, making a total of 180 design points selected.

## RESPONSE SURFACE GENERATION

In this effort, *method i* uses the Response Surface Method (RSM) to find optimal values of $ERE$, $Q_w$, $Q_{inj}$, $W_{rel}$ and $C_{rel}$ for acceptable values of $\Delta P_o$, $\Delta P_f$, $L_{comb}$ and $\Theta$. The approach of RSM is to perform a series of experiments, or numerical analyses, for a prescribed set of design points, and to construct a response surface of the measured quantity over the design space. In the present context, the five responses of interest are $ERE$, $Q_w$, $Q_{inj}$, $W_{rel}$ and $C_{rel}$. The design space consists of the set of relevant design variables $\Delta P_o$, $\Delta P_f$, $L_{comb}$ and $\Theta$. The response surfaces are fit by standard least-squares regression with a quadratic polynomial using the JMP[17] statistical analysis software. JMP is an interactive, spreadsheet-based program which provides a variety of statistical analysis functions. A backward elimination procedure based on t-statistics is used to discard terms and improve the prediction accuracy[18]. When the JMP software is used to analyze the 180 design points, five individual full response surfaces for the variables in the design space are approximated by quadratic polynomials that contain 15 terms each.

In the current study, it is desirable to attempt to maximize $ERE$ and while simultaneously minimizing $Q_w$, $Q_{inj}$, $W_{rel}$ and $C_{rel}$. One method of optimizing multiple responses simultaneously is to build from the individual responses a composite response known as the desirability function. The method allows for a designer's own priorities for the response values to be built into the optimization procedure. The first step in the method is to develop a desirability, $d$, for each response. In the case where a response should be maximized, such as $ERE$, the desirability takes the form:

$$d_1 = \left( \frac{ERE - A}{B - A} \right)^s \tag{4}$$

where $B$ is the target value and $A$ is the lowest acceptable value such that $d = 1$ for any $ERE > B$ and $d = 0$ for $ERE < A$. The power value $s$ is set according to one's subjective impression about

the role of the response in the total desirability of the product. In the case where a response is to be minimized, such as $Q_w$, the desirability takes on the form:

$$d_2 = \left(\frac{Q-E}{C-E}\right)^t \qquad (5)$$

where $C$ is the target value and $E$ is the highest acceptable value such that $d = 1$ for any $Q_w < C$ and $d = 0$ for $Q_w > E$. Choices for $A$, $B$, $C$, and $E$ are chosen according to the designer's priorities or, as in the present study, simply as the boundary values of the domain of $ERE$ and $Q_w$

.

Figure 2. Desirability Function for Various
Weight Factors, $s$.
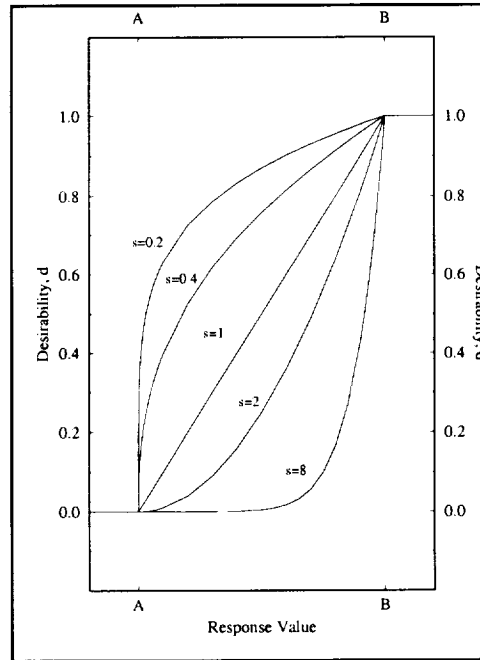


Choices for $s$ and $t$ are more difficult, but plots such as Figure 2 can be instructive. Figure 2 shows the appearance of the desirability function for the case of maximizing a response. Desirabilities with $s<<1$ imply that a product need not be close to the response target value, $B$, to be quite acceptable. But $s = 8$, say, implies that the product is nearly unacceptable unless the response is close to $B$.

A single composite response is developed which is the geometric mean of the desirabilities of the individual responses. The composite response is defined as:

$$D = \left(d_1 \cdot d_2 \cdot d_3 \ldots d_m\right)^{1/m} \qquad (6)$$

The complete joint response surface for the present case is given by:

$$D = \left(d_{ERE} d_{Q_w} d_{Q_m} d_{W_{rel}} d_{C_{rel}}\right)^{1/s} \qquad (7)$$

## OPTIMIZATION RESULTS AND DISCUSION

Two set of results are presented below to demonstrate the capability and flexibility of *method i* for the current injector design. These examples illustrate the effect of each variable on the optimum design and the trade-offs between life and performance issues.

**Effect Of Each Variable On Element Design**

The results in this section were obtained by building the joint response surface with the addition of one dependent variable at a time. The results are shown in Table 1. Case 1 seeks the maximum performance without regard to the effect on the other dependent variables. *ERE* is a fairly strong function of $L_{comb}$—longer chamber lengths allow more residence time for the propellant to mix and burn. The effect of $\Theta$ on *ERE* is strongest at low values of $\Theta$. *ERE* increases with increasing $\Theta$ until about $\Theta = 80°$ and then fall off slightly due to the competing influences noted earlier. These competing influences also cause the effect of both pressure drops on ERE to be somewhat flat, although since $\Delta P_o$ affects more variables, its influence is slightly stronger. Maximum performance is found at high values of $\Delta P_o$, $\Theta$, and $L_{comb}$ and at low values of $\Delta P_f$. This trend is consistent with other works for similar injector elements.[14,15] The value of 98.5 found by the optimizer is indeed the highest predicted by this model. However, since the model developed by Calhoon et al. has been shown to slightly under-predict swirl coaxial element performance, the actual value is likely somewhat higher.

The objective of Case 2 is to simultaneously maximize ERE and the minimize $Q_w$. Table 2 shows that the exact same design point was chosen as for Case 1. Usually, the design which yields the maximum ERE also produces a high wall heat flux. That is not the case here; this issue has already been noted. The minimum $Q_w$ is found in the region of high $\Delta P_o$ and low $\Delta P_f$. In this area, $Q_w$ is almost independent of $\Theta$. Hence, the minimum $Q_w$ can still be found a the high value of $\Theta$ required to maximize ERE. It should be noted that in the low $\Delta P_o$, high $\Delta P_f$ region, $Q_w$ is a function of $\Theta$. Here, as $\Theta$ is increased, $Q_w$ increases since the larger swirl angle forces $d_o$ to increase and thus decrease $V_o$. In the Calhoon et al. model, this reduction in $GO_2$ momentum causes an increase in $Q_w$.

The requirement to minimize $Q_{inj}$ is added in Case 3. In order to minimize $Q_{inj}$, the swirl angle is decreased from 81° to 37°, thus reducing the injector face heat flux by approximately a factor of 3. This decrease in $\Theta$ also lowers *ERE* which forces use of a longer chamber to offset some of the loss. Still, *ERE* is reduced by over one percent.

Case 4 considers the desire to minimize the chamber weight, $W_{rel}$, in addition to maximizing *ERE* and minimizing $Q_w$ and $Q_{inj}$. Since $W_{rel}$ depends only on $L_{comb}$, the chamber length is shortened by over half. The weight goes down, but so does *ERE*. To mitigate the adverse effect on *ERE*, $\Theta$ is increased by almost 10°, simultaneously increasing $Q_{inj}$. *ERE* drops again by over a percent, while $Q_w$ remains constant.

Finally, minimizing the injector cost, $C_{rel}$, is added in Case 5. The relative injector cost is lowered by decreasing each pressure drop approximately a factor of 2. Decreasing $\Delta P_f$ results in a larger fuel gap and decreasing $\Delta P_o$ allows for a larger swirl slot. These factors combine to lower the cost by almost 10 %.

Although several of the variables included in this exercise are qualitative, an important conclusion can still be drawn. The sequential addition of dependent variables to an existing design results in changes to independent and dependent variables in the existing design. The direction and magnitude of these changes depends on the sensitivity of the variables, but the changes may well be significant. The design in Case 5 is quite different that the one in Case 1. Consideration of a larger design space results in a different design—the sooner the additional variables are considered, the more robust the final design.

Table 1. Effect of Each Variable on the Design—Optimal Designs for Original Constraints and Equal Weights.

| Independent Variable | Constraints | Results Case 1 | Results Case 2 | Results Case 3 | Results Case 4 | Results Case 5 |
|---|---|---|---|---|---|---|
| $\Delta P_o$ | 100-200 | 200 | 200 | 200 | 200 | 104 |
| $\Delta P_f$ | 20-200 | 41 | 41 | 42 | 47 | 20 |
| $L_{comb}$ | 2-8 | 7.2 | 7.2 | 7.6 | 3.2 | 3.4 |
| $\Theta$ | 30-90 | 81 | 81 | 37 | 47 | 44 |
| Dependent Variable | Desirability Limits | ERE | ERE & $Q_w$ | ERE, $Q_w$, $Q_{inj}$ | ERE, $Q_w$, $Q_{inj}$, $W_{rel}$ | ERE, $Q_w$, $Q_{inj}$, $W_{rel}$, $C_{rel}$ |
| ERE | 99.3-99.0 | 98.5 | 98.5 | 97.2 | 96.0 | 95.7 |
| $Q_w$ | 0.596-0.647 | 0.596 | 0.596 | 0.596 | 0.596 | 0.596 |
| $Q_{inj}$ | 6.95-36.59 | 26.8 | 26.8 | 9.1 | 12.0 | 10.5 |
| $W_{rel}$ | 0.900-1.154 | 1.13 | 1.13 | 1.14 | 0.97 | 0.98 |
| $C_{rel}$ | 0.73-1.42 | 0.98 | 0.98 | 0.81 | 0.84 | 0.76 |

**Emphasis on Life and Performance Issues**

The purpose of this section is to illustrate the effect of emphasizing certain aspects of the design during the optimization process. *Method i* allows this emphasis via the weights applied to the desirability functions in the joint response surface. The set of results shown in Table 2 facilitate the illustration. The baseline results are repeated in Table 2 from Case 5 in Table 1 where the entire design space is considered using the original constraints and equal weights for the dependent variables. The results in for Case 1 are obtained by emphasizing the minimization of the wall and injector face heat fluxes. Desirability functions for both of these variables are given increased weights (5 and 10, respectively). Since lower heat fluxes tend to increase component life, weighting these two variables is equivalent to emphasizing a life-type issue in the design. $Q_w$ is already at its minimum value. As expected, $\Theta$ is decreased which decreases the value of $Q_{inj}$ by almost 35%. The lower value of $\Theta$ also produces a lower *ERE*. Both propellant pressure drops and the combustor length are increased to mitigate the drop in *ERE*. The increases in $L_{comb}$ and $\Delta P_f$ cause increases in $W_{rel}$ and $C_{rel}$, respectively. The emphasis on life extracts the expected penalty on performance. Additionally, for the current model, there are also slight weight and cost penalties.

The results for Case 2 are obtained by emphasizing maximization of *ERE* and minimization of $W_{rel}$ with desirability weightings of 10 and 5, respectively. Increased weighting for these two variables is equivalent to emphasizing a thrust to weight goal for the injector/chamber. The relative chamber length is shortened to slightly lower $W_{rel}$. *ERE* is maximized by increasing the GO2 swirl angle by a factor of almost 2.5 and also increasing $\Delta P_f$ by over 35 %. The value of *ERE* rises by over one percent. As noted earlier, increasing $\Theta$ leads to increased injector heat flux. For this case, emphasis on thrust and weight tends to have an adverse affect on $Q_{inj}$. Relative cost, for the current model, is also increased significantly.

Table 2. Effect of Emphasizing Life and Performance Issues.

| Independent Variable | Constraints | Results Baseline | Constraints | Results Case 1 | Constraints | Results Case 2 |
|---|---|---|---|---|---|---|
| $\Delta P_o$ | 100-200 | 104 | 100-200 | 200 | 100-200 | 200 |
| $\Delta P_f$ | 20-200 | 20 | 20-200 | 32 | 20-200 | 44 |
| $L_{comb}$ | 2-8 | 3.4 | 2-8 | 3.6 | 2-8 | 2.9 |
| $\Theta$ | 30-90 | 44.0 | 30-90 | 30.0 | 30-90 | 72.0 |
| Dependent Variable | *Baseline Variable Weight* | | *Life Variable Weight* | | *Thrust/Weight Variable Weight* | |
| ERE | 1 | 95.7 | 1 | 95.3 | 10 | 96.7 |
| $Q_w$ | 1 | 0.596 | 5 | 0.596 | 1 | 0.596 |
| $Q_w$ | 1 | 10.5 | 10 | 6.9 | 1 | 22.6 |
| $W_{rel}$ | 1 | 0.98 | 1 | 0.99 | 2 | 0.96 |
| $C_{rel}$ | 1 | 0.76 | 1 | 0.79 | 1 | 0.94 |

## SUMMARY

A swirl coaxial $GO_2/GH_2$ injector element design has been employed to facilitate optimization studies. Starting with propellant pressure drops, combustor length, and full cone swirl angle, an empirical design methodology was used to calculate the dependent variables for 180 design points. The dependent variables were energy release efficiency, chamber wall and injector face heat fluxes, relative chamber weight, and relative injector cost. The response surface methodology was used to fit the results with quadratic polynomials. Desirability functions were used to create joint response surfaces which were used in the optimization studies.

Two sets of results were generated to illustrate the capability of *method i* in the context of injector design and optimization. The first set of results started with a design optimized for *ERE* and then added the other four dependent variables to the design one at a time. Most sequential optimal designs were different than previous designs, with the final design being quite different than the initial design. The result showed the importance of including as many variables as possible early in the design. The optimization techniques embodied in *method i* facilitate this early inclusion by allowing efficient management of large amounts of data.

The second set of results focused on the inherent design trade-offs between performance and component life. Different weights were applied to emphasize variables related to performance (*ERE* and $W_{rel}$). While the thrust to weight ratio was improved, the adverse affect on variables

related to component life ($Q_w$ and $Q_{inj}$) were clearly shown. Conversely, when $Q_w$ and $Q_{inj}$ were emphasized, the toll on the performance variables was clear. These techniques can be used to identify both qualitative trends and to examine the quantitative trade-offs present in this and other design processes.

The flexibility and utility of *method i* have been demonstrated in this effort. Use of *method i* can allow an injector designer to confidently and efficiently manage large amounts of data to conduct a range of design optimization studies. Constraints on independent variables can be modified to allow optimum designs to be sought in specific portions of the parameter space. Also, individual or specific groups of dependent variables can be emphasized to reflect a designer's priorities in the design optimization process.

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